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Auto-localization in de-Sitter space

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Abstract: We point out that gravity on dS_n gives rise to a localized graviton on dS_{n-1}. This way one can derive a recursion relation for the entropy of dS spaces, which might have interesting implications for dS holography. In the same spirit we study domain walls interpolating between dS spaces with different cosmological constant. Our observation gives an easy way to calculate what fraction of the total entropy can be accessed by an observer stuck on the bubble wall.

Keywords: AdS-CFT and dS-CFT Correspondence, Classical Theories of Gravity.

Contents

1. Counting degrees of freedom in compactifications

Consider a higher dimensional, or to be specific: a 10-dimensional space-time of the product form $dS_5 \times M_5$ where M_5 is a compact space of volume V and dS_5 is a de-Sitter space of curvature radius L. At long distances gravity can be described by an effective $5d$ theory, where the effective $5d$ Planck scale is given by

$$M^3_{pl,5d} = V M^8_{pl,10d} \, . \tag{1.1}$$

This spacetime has an entropy which is given by

$$S = \frac{1}{4} \mathcal{A}_3 L^3 V M^8_{pl,10d} \, , \tag{1.2}$$

where

$$\mathcal{A}_n = 2 \frac{\pi^{\frac{n+1}{2}}}{\Gamma[\frac{n+1}{2}]} \tag{1.3}$$

denotes the volume of the unit n-sphere. The formula can be derived in two ways: in the full microscopic geometry $\mathcal{A}_3 L^3 V$ is the horizon area. In the effective long distance description, $\mathcal{A}_3 L^3$ is the usual $5d$ dS horizon area in the theory with effective Planck scale $V M^8_{pl,10d}$. So both in $10d$ and in $5d$ the entropy is the horizon area in Planck units.

One way to understand why the effective $5d$ counting has to reproduce the right number, is that according to rather general argument, see e.g. [1] (or see [2] for a review), the area of the horizon in Planck units is really a bound on the total number of degrees of freedom accessible to any quantum theory containing gravity in a dS spacetime. The $10d$ theory on $dS_5 \times M_5$ is just a particular realization of a quantum theory on dS_5 containing the massless graviton and in addition a tower of massive KK spin 2 particles. So the entropy in the full "microscopic" $10d$ description has to agree with number derived from the $5d$ horizon area in $5d$ Planck units.

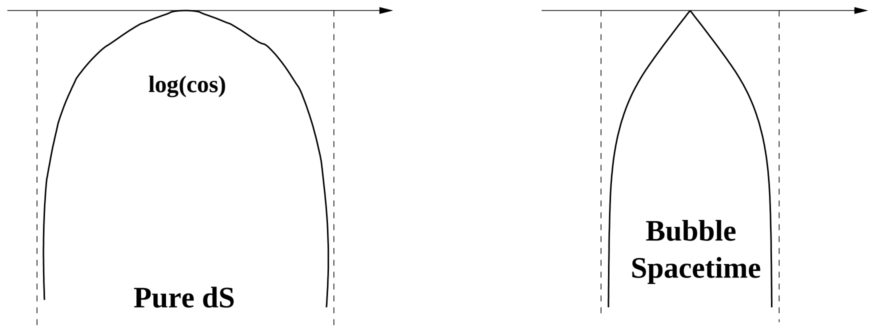

Figure 1: Warpfactor for the de Sitter slicing of de Sitter space. Even without any brane or defect we get a localized, normalizable zero mode. Including a brane as usual pastes together two regions of pure de Sitter.

Indeed we expect this counting to still be true even if the volume V of M_5 is of the same order as the curvature radius L of the dS space. In this case $5d$ gravity would not be a good description to describe the force law an observer in the dS_5 space would measure for test masses that a separated by distance smaller than the size of the horizon. However the usual arguments still suggest that the accessible number of degrees of freedom in this theory containing gravity and a tower of light spin-2 KK modes are still finite and given by the $5d$ horizon area. We will show that in the same spirit one can derive an amusing recursion relation between the entropy of dS spaces of various dimensions. The formalism developed can also be used to gain some insight into the entropy of bubble spacetimes.

2. Auto-localization in dS space

In this section we are going to establish that gravity in pure dS space can be viewed as lower dimensional dS gravity coupled to a KK-mode matter system. In order to see a localized graviton in a mechanism similar to the one employed by RS [3], we first need to write dS as a warped product. It turns out that the only allowed spatial slicing in terms of maximally symmetric codimension one spacetimes is a dS slicing

$$ds^2_{dS_d} = e^{2A(r)} ds^2_{dS_{d-1}} + dr^2\,, \tag{2.1}$$

where dS_{d-1} has curvature radius $1/\sqrt{\Lambda}$. The corresponding warpfactor is given by

$$A = \log\left(\sqrt{\Lambda} L \cos\frac{r}{L}\right). \tag{2.2}$$

Fixing $A(0) = 0$ yields $\Lambda = 1/L^2$, the dS_{d-1} radius at the turn around point is the same as the dS_d radius. Go to conformal form

$$ds^2_{dS_d} = e^{2A(z)}\left(ds^2_{dS_{d-1}} + dz^2\right) \tag{2.3}$$

with

$$\begin{aligned} z(r) &= \frac{1}{\sqrt{\Lambda}} \operatorname{arccosh}\left(\frac{1}{\cos(r/L)}\right) \\ e^{A(z)} &= \frac{1}{\cosh(z\sqrt{\Lambda})}\,. \end{aligned} \tag{2.4}$$

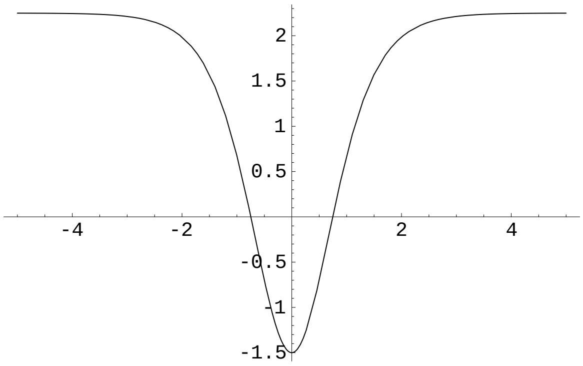

Figure 2: Volcano potential for the dS slicing of pure dS. This (and the other potentials shown) is plotted for $d = 5$ and $L = 1$.

Using the formalism and conventions of [4] we can translate the differential equations of linearized gravity into an analog quantum mechanics with volcano potential

$$V(z) = \frac{(d-2)^2}{4} A''(z) + \frac{d-2}{2} A'(z)^2 = \Lambda \left(\frac{(d-2)^2}{4} - \frac{d-2}{4} d \frac{1}{\cosh^2(z\sqrt{\Lambda})} \right). \tag{2.5}$$

This potential has a single bound state at zero energy, separated from a continuum of modes separated by a mass gap of order $1/L^2$. The zero mode solution

$$\psi = e^{\frac{d-2}{2} A(z)} = \left(\cosh(z\sqrt{\Lambda}) \right)^{\frac{2-d}{2}} \tag{2.6}$$

is a normalizable solution to the wave equation with $E = 0$. So the full theory can be rewritten in terms of a $d-1$ dimensional graviton on the slice at $r = 0$.

To relate the d-dimensional Planck scale to the $d-1$ dimensional Planck scale we can use the fact that the norm of the zero mode is effectively the volume of the internal space. Evaluating the norm

$$V \equiv \int dz |\psi(z)|^2 = \sqrt{\pi} \frac{\Gamma(\frac{d}{2} - 1)}{\Gamma(\frac{d-1}{2})} L \tag{2.7}$$

we get

$$M_{Pl,d-1}^{d-3} = V \; M_{Pl,d}^{d-2} = \frac{A_{d-2}}{A_{d-3}} \; L \; M_{Pl,d}^{d-2} \tag{2.8}$$

and one can check that indeed the entropy of the dS_{d-1} using the Planck scale of the localized graviton is identical to the entropy of the dS_d using its original Planck scale:

$$4S = A_{d-2} \; M_{Pl,d}^{d-2} \; L^{d-2} = A_{d-3} \; M_{Pl,d-1}^{d-3} \; L^{d-3} \, . \tag{2.9}$$

Obviously the theory is not really lower dimensional. Even though the continuum modes are separated by a mass gap, they contribute at length scales smaller than the horizon radius, since their mass is set by the dS radius. But still, as in the case of a compactification, arguing that any theory of gravity in de Sitter space, even a theory containing a continuum of light spin-2 modes, should not exceed the entropy of the horizon area in Planck units, we can reproduce the exact entropy in terms of the localized graviton.[1]

A note on holography: so far we analyzed the purely classical theory of gravity fluctuations in dS_d space. We found that they can be described in terms of a localized graviton on the central dS_{d-1} slice coupled to a KK-continuum that is separated from the zero mode by a mass-gap of order the inverse dS curvature radius. In the similar situation in AdS it has been established that the KK continuum can be holographically replaced by a $d-1$ dimensional quantum field theory, still coupled to the classical localized graviton [5]–[8]. If this would be possible here as well, repeated application would provide us with a way to replace gravity in any dS by gravity in a $2d$ dS space (which recently has been shown to be a tractable theory even on the quantum level [9, 10]) coupled to a complicated quantum field theory representing all the KK modes. Eventually this $2d$ system could even be replaced by a matrix model. This might be the best way to get a handle on holography in dS spaces.

A different proposal has been advanced in the past, known as the dS/CFT correspondence [11, 12] (see also [13, 14]), where the holographic dual of dS gravity is a euclidean CFT living at the boundary at future or past infinity. The validity of this picture has been questioned by many recent papers, in particular [15]–[17]. In this language our decomposition would correspond to the euclidean version of the AdS/dCFT story of [18]–[21]. In this language what we described above is the analog of the "halfway dual" of [8, 22], where only the KK-modes get replaced by a quantum CFT coupled to the classical graviton localized on the brane. In dS latter might prove to be the more powerful statement.

3. dS domain walls

It is easy to see what happens when we include domain walls with a non-zero positive tension λ. They live at an $r = const.$ slice such that

$$\lambda \propto A'|_{\mathrm{Left}} - A'|_{\mathrm{Right}} \,. \tag{3.1}$$

These domain walls have received some recent attention [23]. If one allows the cosmological constant to jump across the brane (give them charge under a background d-form fieldstrength), they correspond to the bubbles of false vacuum decay. In this section we will focus on the simplest case of a neutral brane. For earlier studies of localization of gravity on dS branes in dS see [24]–[28].

A zero tension "phantom brane" lives at $r = 0$, that is, as we have seen above, without any brane pure de-Sitter localizes a graviton on a de-Sitter slice in the "center" of dS. At finite tension the brane lives at a finite r_0 (or z_0 in the conformal coordinates) and the

[1] One way to see why the integral had to work is that the dS slicing of dS gets inherited after Wick rotation from the usual way to write the n-sphere in terms of $n-1$ spheres:

$$ds^2_{S^n} = \cos^2(\theta) ds^2_{S^{n-1}} + d\theta^2 \,. \tag{2.10}$$

Evaluating the volume of this space leads to precisely the same integral over theta we used to evaluate the norm above.

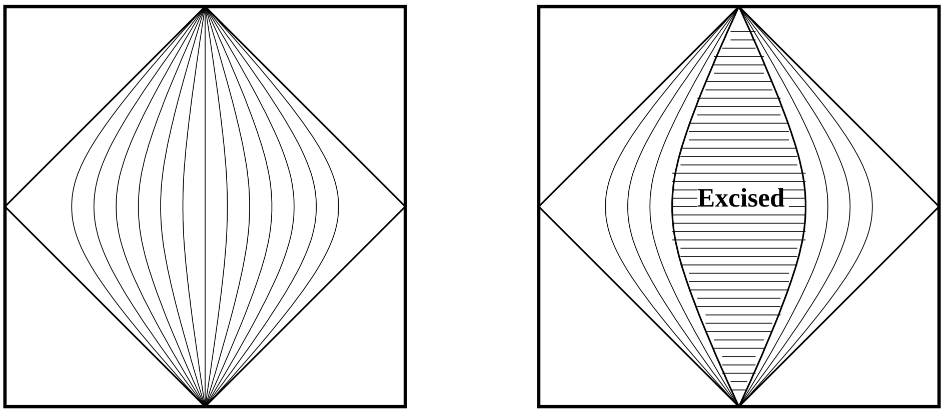

Figure 3: Embedding of the dS slicing in dS. The brane cuts out the shaded region in the diagram on the right.

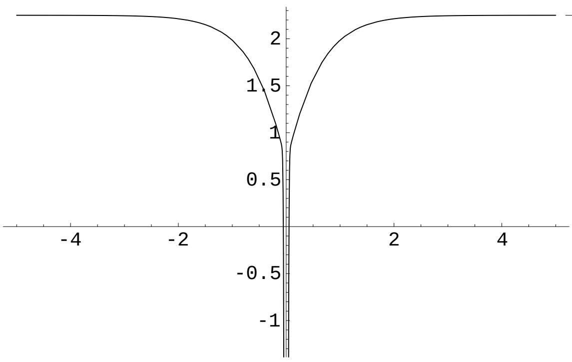

Figure 4: Volcano potential for $z_0 = 1$ at $L = 1$, $d = 5$.

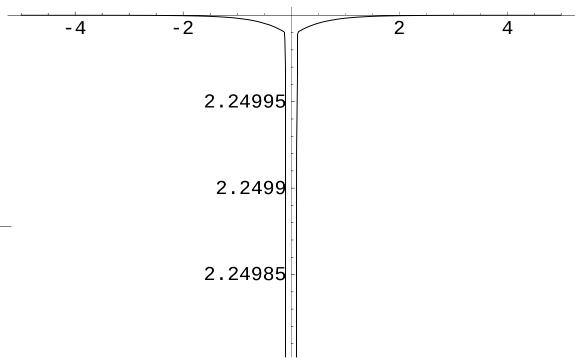

Figure 5: Volcano potential for $z_0 = 7$ at $L = 1$, $d = 5$.

region with $|r| < r_0$ gets excised from space-time. The effective cosmological constant on the brane is given by:

$$\frac{1}{L_{d-1}^2} = \frac{1}{L^2} + \frac{1}{l^2}\,, \tag{3.2}$$

where l is the curvature radius associated with the cosmological constant due to the brane tension λ.

Volcano potentials for branes with increasing tension are depicted in figures 4 and 5. The structure is always the same: a single boundstate zero mode separated from a continuum by a mass gap

$$M_{\rm gap}^2 = \frac{(d-2)^2}{4L_{d-1}^2}\,. \tag{3.3}$$

Note that it is the curvature radius of the brane dS space that enters. In the extreme case of a dS_{d-1} brane in Minkowski space we just have the usual delta function in an otherwise flat potential first studied in [29].

It is interesting to ask, what happens to the entropy. As the brane moves out to finite values of r, the curvature radius of the lower-dimensional dS decreases with the warpfactor.

But at the same time the volume of space shrinks. The entropy of the localized graviton on dS_{d-1} is:

$$4S = A_{d-2} M_{Pl,d}^{d-3} V L_{d-1}^{d-3}\,, \tag{3.4}$$

where (setting $|\Lambda = 1/L^2|$)

$$L_{d-1} = e^{A(z_0)} L < L \tag{3.5}$$

and

$$V = \int_{z_0}^{\infty} |\psi(z)|^2 < V_0 = \int_0^{\infty} |\psi(z)|^2\,. \tag{3.6}$$

That is, the entropy of the localized graviton is bounded to be strictly less than the entropy of the higher dimensional de Sitter space! Naively one would have expected that the entropy of the excised region gets stored in a holographic fashion as matter entropy on the brane. But lower dimensional gravity predicts that the brane entropy is less than the entropy of the original de Sitter space. How shall we interpret this result? The entropy we calculated is the entropy accessible to an observer on the brane. As usual in dS space, parts of the brane will be hidden behind horizons. This entropy has to be distinguished from the full "open string" entropy carried by the brane, which featured prominently in [23]. There are bulk observers who can access the full entropy of degrees of freedom on the brane [23]. We have nothing new to say about the latter.

In the extreme case that the brane tension becomes much larger than the bulk cc we recover the case of a dS brane in flat space. In this case the bulk entropy goes to infinity when we keep the brane entropy finite.

4. Newton law on the brane?

All we needed for our discussion of the previous sections was to have a localized, normalizable zero mode in order to apply the lower dimensional holographic bound. One may wonder whether in any of the cases discussed above, gravity looks genuinely lower dimensional. This is never the case. Note that even though all the KK-modes are massive, the gap is always of order $1/L_{d-1}$. So the KK-modes can only be neglected at distance scales larger than the horizon radius of the lower dimensional dS space. At length scales accessible to any observer on the brane, we always see the effect of the light, massive KK continuum modes. Never do we get a lower dimensional Newton law. But let's emphasize again, that this does not invalidate the entropy counting in terms of the lower dimensional dS space.

In the same spirit as above we can similarly study domain walls across which the bulk cc jumps, that is they are charged under a d-form field strength. We are not restricted to having jumps from dS into dS only, we can as well have jumps from dS into Minkowski or AdS. In this case the warpfactor is $\log(r)$ or $\log(\sinh(r))$ respectively instead of the $\log(\cos(r))$. One can always find solutions that paste those together like in the right diagram of figure 1, that is in the "up-down" fashion.

The situation becomes more interesting when one considers "up-up" spacetimes. Those are only possible for dS/dS bubbles. This happens when the brane has charge but almost no tension. The warpfactor than looks very close to the left diagram of figure 1 with a

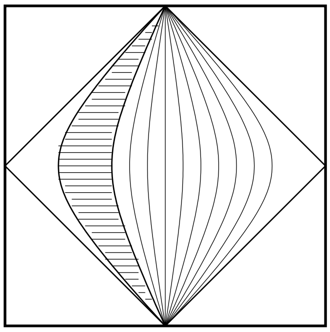

Figure 6: Almost tensionless bubble interpolating between two dS spaces with different cosmological constant.

brane of center inducing a little additional jump. In this scenario the localized graviton is not localized on the brane but it is localized in the center of dS as in the no-brane case. The dS radius of the localizing slice is hence always L, even as L_{d-1} can become very small.

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